高次方程式の解

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2020 年 3 月 17 日

1　目的

ベアストウ法により，$a_n x^n + a_{n-1} x^{n-1} + \cdot + a_2 x^2 + a_1 x + a_0 = 0$ の解を求める。虚数解も含め n 個の解全てを求める。

2　使用法

```
import sys
sys.path.append("statlib")
from misc import Bairstow, quadratic_root
Bairstow(a, EPSILON = 1e-14)
quadratic_root(p, q)
```

2.1　引数

a	最高次数から定数項までの係数ベクトル
EPSILON	精度（デフォルトは 10^{-14}）
p，q	2 次式の係数，ただし，1 次の項の係数と定数項

2.2　戻り値

求められた解

3　使用例

sympy パッケージの solve() での解と比較するための準備

```
from sympy import *
var('x')
```

x

```
def sympy_solve(a):
    order = len(a)-1
```

```
    expr = '+'.join([str(j)+"* x**"+str(order-i) for i, j in enumerate
        (a)])
    ans = solve(expr)
    return np.array([x.evalf() for x in ans])

import sys
sys.path.append("statlib")
from misc import Bairstow, quadratic_root
```

```
import numpy as np
Bairstow(np.array([1, -15, 85, -225, 274, -120]))
```

```
array([1., 2., 3., 4., 5.])
```

```
sympy_solve([1, -15, 85, -225, 274, -120])
```

```
array([1.00000000000000, 2.00000000000000, 3.00000000000000,
       4.00000000000000, 5.00000000000000], dtype=object)
```

```
Bairstow([1, 10, 35, 50, 24])
```

```
array([-4., -3., -2., -1.])
```

```
sympy_solve([1, 10, 35, 50, 24])
```

```
array([-4.00000000000000, -3.00000000000000, -2.00000000000000,
       -1.00000000000000], dtype=object)
```

```

Bairstow([1, 10, 25, 50, 24])
```

```
array([-7.49826796+0.j        , -0.93451222-2.04584549j,
       -0.93451222+2.04584549j, -0.63270759+0.j        ])
```

```
sympy_solve([1, 10, 25, 50, 24])
```

```
array([-0.632707591668536, -0.934512223227342 - 2.04584548724792*I,
       -0.934512223227342 + 2.04584548724792*I, -7.49826796187678],
      dtype=object)
```

```

Bairstow([1, 1, 1, 1])
```

```
array([-1.+0.j,  0.-1.j,  0.+1.j])
```

```
sympy_solve([1, 1, 1, 1])
```

```
array([-1.00000000000000, -1.0*I, 1.0*I], dtype=object)
```

```
Bairstow([1, 1, 1])
```

```
array([-0.5-0.8660254j, -0.5+0.8660254j])
```

```
sympy_solve([1, 1, 1])
```

```
array([-0.5 - 0.866025403784439*I, -0.5 + 0.866025403784439*I],
      dtype=object)
```

```
Bairstow([3, 5])
```

```
array([-1.66666667])
```

```
sympy_solve([3, 5])
```

```
array([-1.66666666666667], dtype=object)
```

```
Bairstow([1, -2, 1])
```

```
array([1., 1.])
```

```
sympy_solve([1, -2, 1])
```

```
array([1.00000000000000], dtype=object)
```

```
quadratic_root(-1.000000001, 0.000000001)
```

```
(1.0, 1e-09)
```

```
quadratic_root(0, 1)
```

```
(1j, -1j)
```

```
Bairstow([1,0,4,0,4]) # +/- sqrt(2) i
```

```
array([-1.17315892e-09-1.41421356j, -1.17315892e-09+1.41421356j,
        1.17315892e-09-1.41421356j,  1.17315892e-09+1.41421356j])
```

```
sympy_solve([1,0,4,0,4])
```

```
array([-1.4142135623731*I, 1.4142135623731*I], dtype=object)
```

```
Bairstow([1,0,0,0,-12,0,-16])
```

```
array([-2.00000000e+00+0.j        , -6.56126664e-23-1.41421357j,
       -6.56126664e-23+1.41421357j,  6.31339519e-23-1.41421356j,
        6.31339519e-23+1.41421356j,  2.00000000e+00+0.j        ])
```

```
sympy_solve([1,0,0,0,-12,0,-16])
```

```
array([-2.00000000000000, 2.00000000000000, -1.4142135623731*I,
       1.4142135623731*I], dtype=object)
```

```
Bairstow([1,0,-2,0,1])
```

```
array([-1., -1.,  1.,  1.])
```

```
sympy_solve([1,0,-2,0,1])
```

```
array([-1.00000000000000, 1.00000000000000], dtype=object)
```

```
Bairstow([1,0,4,0,5,0,2]) # +/- 1i, +/- sqrt(2) i
```

```
array([-1.01118417e-15-1.j        , -1.01118417e-15+1.j        ,
        3.69141465e-24-1.41421356j,  3.69141465e-24+1.41421356j,
        1.01118416e-15-1.j        ,  1.01118416e-15+1.j        ])
```

```
sympy_solve([1,0,4,0,5,0,2])
```

```
array([-1.0*I, 1.0*I, -1.4142135623731*I, 1.4142135623731*I], dtype=object)
```

```
Bairstow([1,-1,7,-6,18,-12,20,-8,8]) # +/- sqrt(2) i, 0.5 +/- sqrt
    (3)/2 i
```

```
array([-3.13733711e-06-1.41421674j, -3.13733711e-06+1.41421674j,
       -1.18187354e-06-1.41420926j, -1.18187354e-06+1.41420926j,
        4.31921065e-06-1.41421469j,  4.31921065e-06+1.41421469j,
        5.00000000e-01-0.8660254j ,  5.00000000e-01+0.8660254j ])
```

```
sympy_solve([1,-1,7,-6,18,-12,20,-8,8])
```

```
array([-1.4142135623731*I, 1.4142135623731*I, 0.5 - 0.866025403784439*I,
       0.5 + 0.866025403784439*I], dtype=object)
```

```
Bairstow([1,3,3,1]) # -1 [3]
```

```
array([-1.00000423+0.00000000e+00j, -0.99999789-3.66006312e-06j,
       -0.99999789+3.66006312e-06j])
```

```
sympy_solve([1,3,3,1])
```

```
array([-1.00000000000000], dtype=object)
```

```
Bairstow([1,-1,1,-1,-2,2]) # 1 [2], -1, +/- sqrt(2) i
```

```
array([-1.00000000e+00+0.j        ,  7.70279857e-16-1.41421356j,
        7.70279857e-16+1.41421356j,  9.99999995e-01+0.j        ,
        1.00000001e+00+0.j        ])
```

```
sympy_solve([1,-1,1,-1,-2,2])
```

```
array([-1.00000000000000, 1.00000000000000, -1.4142135623731*I,
       1.4142135623731*I], dtype=object)
```

```
Bairstow([1,-1,6,-5,13,-8,12,-4,4]) # +/- sqrt(2) i [2], 0.5 +/- sqrt
    (3)/2 i, +/- i
```

```
array([-1.14138115e-09-1.41421359j, -1.14138115e-09+1.41421359j,
       -4.59252412e-17-1.j        , -4.59252412e-17+1.j        ,
        1.14138090e-09-1.41421354j,  1.14138090e-09+1.41421354j,
        5.00000000e-01-0.8660254j ,  5.00000000e-01+0.8660254j ])
```

```
sympy_solve([1,-1,6,-5,13,-8,12,-4,4])
```

```
array([-1.0*I, 1.0*I, -1.4142135623731*I, 1.4142135623731*I,
       0.5 - 0.866025403784439*I, 0.5 + 0.866025403784439*I], dtype=object)
```

```
Bairstow([1,-2,3,-2,1]) # 0.5 +/- sqrt(3)/2 i [2]
```

```
array([0.49999999-0.8660254j, 0.49999999+0.8660254j,
       0.50000001-0.8660254j, 0.50000001+0.8660254j])
```

```
sympy_solve([1,-2,3,-2,1])
```

```
array([0.5 - 0.866025403784439*I, 0.5 + 0.866025403784439*I], dtype=object)
```

```
Bairstow([12,12,24,24,0,0]) # 0 [2], -1, +/- sqrt(2) i
```

```
array([-1.+0.j        ,  0.-1.41421356j,  0.+0.j        ,  0.+0.j        ,
        0.+1.41421356j])
```

```
sympy_solve([12,12,24,24,0,0])
```

```
array([-1.00000000000000, 0, -1.4142135623731*I, 1.4142135623731*I],
      dtype=object)
```

```
Bairstow([1,1,2,2])
```

```
array([-1.+0.j        ,  0.-1.41421356j,  0.+1.41421356j])
```

```
sympy_solve([1,1,2,2])
```

```
array([-1.00000000000000, -1.4142135623731*I, 1.4142135623731*I],
      dtype=object)
```

ベアストウ法も，重解がある場合には計算に失敗することがある。
そんな場合でも，solve() は正しく解を求めている。

```
factor(x**4 + 2*x**3 + 3*x**2 + 2*x + 1)
```

$\left(x^2 + x + 1\right)^2$

```
Bairstow([1,2,3,2,1]) # -0.5 +/- sqrt(3)/2 i
```

```
statlib/misc.py:159: RuntimeWarning: invalid value encountered in double_scalars
  delta_p = (b[n - 1] * c[n - 2] - b[n] * c[n - 3]) / d
statlib/misc.py:160: RuntimeWarning: invalid value encountered in double_scalars
  delta_q = (b[n] * c[n - 2] - b[n - 1] * (c[n - 1] - b[n - 1])) / d
ill condition. p = nan   q = nan   d = nan
```

```
---------------------------------------------------------------------------Exception
----> 1 Bairstow([1,2,3,2,1]) # -0.5 +/- sqrt(3)/2 i
statlib/misc.py in Bairstow(a, EPSILON)
    165         else:
    166             print("ill condition. p =", p, "  q =", q, "  d =", d)
--> 167             raise Exception("abnormal termination.")
    168         ans.extend(quadratic_root(p, q))
```

```
    169         a = b[:n - 1]
Exception: abnormal termination.
```

```
sympy_solve([1,2,3,2,1])
```

```
array([-0.5 - 0.866025403784439*I, -0.5 + 0.866025403784439*I],
      dtype=object)
```

```
factor(x**7 + x**6 + x**5 - x**4 - x**3 - x**2)
```

$$x^2 (x-1) \left(x^2+x+1\right)^2$$

```
Bairstow([1,1,1,-1,-1,-1,0,0]) # 0 [2], 1, -0.5 +/- sqrt(3)/2 i [2]
```

```
ill condition. p = nan   q = nan   d = nan
statlib/misc.py:159: RuntimeWarning: invalid value encountered in double_scalars
  delta_p = (b[n - 1] * c[n - 2] - b[n] * c[n - 3]) / d
statlib/misc.py:160: RuntimeWarning: invalid value encountered in double_scalars
  delta_q = (b[n] * c[n - 2] - b[n - 1] * (c[n - 1] - b[n - 1])) / d

---------------------------------------------------------------------------Exception
----> 1 Bairstow([1,1,1,-1,-1,-1,0,0]) # 0 [2], 1, -0.5 +/- sqrt(3)/2 i [2]
statlib/misc.py in Bairstow(a, EPSILON)
    165         else:
    166             print("ill condition. p =", p, "  q =", q, "  d =", d)
--> 167             raise Exception("abnormal termination.")
    168         ans.extend(quadratic_root(p, q))
    169         a = b[:n - 1]
Exception: abnormal termination.
```

```
sympy_solve([1,1,1,-1,-1,-1,0,0])
```

```
array([0, 1.00000000000000, -0.5 - 0.866025403784439*I,
       -0.5 + 0.866025403784439*I], dtype=object)
```

```
factor(x**8 + 2*x**6 + 3*x**4 + 2*x**2 + 1)
```

$$\left(x^2-x+1\right)^2 \left(x^2+x+1\right)^2$$

```
Bairstow([1,0,2,0,3,0,2,0,1]) # +/- 0.5 +/- sqrt(3)/2 i 「2」
```

```
statlib/misc.py:159: RuntimeWarning: invalid value encountered in double_scalars
  delta_p = (b[n - 1] * c[n - 2] - b[n] * c[n - 3]) / d
statlib/misc.py:160: RuntimeWarning: invalid value encountered in double_scalars
  delta_q = (b[n] * c[n - 2] - b[n - 1] * (c[n - 1] - b[n - 1])) / d
ill condition. p = nan   q = nan   d = nan
```

```
---------------------------------------------------------------------------Exception
----> 1 Bairstow([1,0,2,0,3,0,2,0,1]) # +/- 0.5 +/- sqrt(3)/2 i「2」
statlib/misc.py in Bairstow(a, EPSILON)
    165         else:
    166             print("ill condition. p =", p, "  q =", q, "  d =", d)
--> 167             raise Exception("abnormal termination.")
    168         ans.extend(quadratic_root(p, q))
    169         a = b[:n - 1]
Exception: abnormal termination.
```

```
sympy_solve([1,0,2,0,3,0,2,0,1])
```

```
array([-0.5 - 0.866025403784439*I, -0.5 + 0.866025403784439*I,
       0.5 - 0.866025403784439*I, 0.5 + 0.866025403784439*I], dtype=object)
```